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DESIGN AND ANALYSIS OF A DISCRETE MODULAR SERPENTINE ROBOTIC TAIL FOR IMPROVED PERFORMANCE OF MOBILE ROBOTS

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ABSTRACT

This paper presents the design and analysis of a novel Discrete Modular Serpentine Tail for mobile legged robotic systems. These systems often require an inertial appendage to generate forces and moments to provide a means of improved performance in terms of stabilization, maneuvering and dynamic self-righting in addition to enhancing manipulation capabilities. The majority of existing tail designs consist of planar pendulums that limit improved performance to specific planes due to limited articulation. The proposed system consists of a modular two degree of freedom, spatial mechanism constructed from rigid segments actuated by cable tension and displacements whose curvatures are dependent on a multi-diameter pulley. Modules can be interconnected in series to achieve multiple spatial curvatures; thus, can bring about multi-planar improved performance and enhanced manipulation capabilities to the overall robotic system. First, the detailed design is presented after which the forward kinematics of the mechanism is derived to analyze both the kinematic coupling between the segments and the influence between the ratio of segment lengths and pulley diameters. The equations of motion are derived and modified due to cable tension driving segments and are used to determine torque requirements of the system to aid the design process for motor selection. Multi-objective optimal kinematic synthesis is then formulated and presented as a case study for synthesizing physical dimensions of the mechanism to achieve the best fit of user defined tail curvatures.

1 INTRODUCTION

By observing nature's abundant store of solutions, engineers can gain a source of inspiration in designing robots that are referred to as "bio-inspired" products to tackle major challenges persisting within the field of robotics. However, one aspect that has not been thoroughly explored is the incredible way animals use their articulated tails for a variety of tasks ranging from stabilization, maneuvering, dynamic self-righting and manipulation and its practical implementation to mimic these tasks in the field of mobile robotics.

An extensive review of the usage of tails in the animal kingdom has been provided by Hickman [1]. Kangaroos have been observed to use their tails as a counter balance anchor point to provide a stable posture while standing on their hind legs [2]. They are also known to swing their tails at high speeds to reorient their bodies in mid-air. Similarly, dinosaurs such as the Tyrannosaurus Rex were believed to use their tails as a means of stabilization during a walking gait to maintain the yaw of their body [3, 4]. Lizards have also been observed in using their tails for aerial maneuvers to adjust their pose for smooth landing [5, 6].

Only a few robots have utilized tails for more than aesthetic purposes. Unirrro [7] was one of the first legged robot to utilize a tail capable of pitch motions to cancel hoping motion and maintain constant body pitch. A similar tail design was implemented on both a lizard [8] and kangaroo [9] inspired robot for mid-air attitude control and self-righting. Kohut et al. demonstrated turning of a 44g legged robot utilizing a tail capable of yaw motion [10, 11]. A Cheetah inspired tail was used for attitude control, airborne maneuvers and disturbance rejection [12]. Although these robots are bio-inspired, their tails are single degree of freedom rigid pendulums that limited improved performance to specific planes and were not used for tasks involving manipulation due to their limited articulation.

To address these issues, this research aims at developing a novel, agile robotic tail design that can provide multi-planar performance improvements. The analysis in this paper will aid the design and kinematic synthesis of robotic serpentine tails, and the selection of actuators to achieve desired trajectory profiles. This paper is part of ongoing research to contribute to

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the field of robotic tail design and its practical implementation onto mobile legged robots [13-19].

The paper is organized as follows: Section 2 provides an overview of existing robotic structures that have inspired the design of the DMST. Section 3 presents the mechanical design of the proposed mechanism. Section 4 derives the forward kinematics of the mechanism to analyze both the kinematic coupling between the segments and the influence between the ratio of segment lengths and pulley diameters. Section 5 presents dynamic analysis where equations of motion of the system are derived and used to calculate torque requirements of the system. Section 6 presents the formulation of multi-objective optimal kinematic synthesis problem to determine physical dimensions of the mechanism to achieve a best fit of user defined tail curvatures. Concluding remarks and future work are discussed in Section 7.

2 BACKGROUND

This section provides an overview of articulate mechanisms in the field of continuum and serpentine robotics to: 1) identify the design challenges and 2) highlight useful design criteria that has inspired the development of the DMST.

Continuum robots consist of an elastic backbone core, capable of forming continuous curves in space similar to tentacles, tongues, and tails found in the animal kingdom [20]. These robots are often classified according to their method of actuation [21]. Intrinsic designs utilize actuators located on and form part of the robotic structure while extrinsic designs use remote actuation often in the form of cables or rods to transfer motion [22]. Extrinsic actuation provides the benefits of smaller cross section and lighter weight designs that closer resemble biological limbs where displacement is produced by pulling tendons. Although continuum robots are capable of forming articulate spatial configurations, elasticity induces sagging due to gravitational loading that poses difficulties in modeling kinematics and sensing the robots configuration [14, 16].

Serpentine robots are rigid link robotic systems that are composed of a large number of rigid-modules (often identical) capable mimicking the continuous curvatures of their biological counterparts with discrete bending motions [23-25]. They are respectively distinguished from conventional serial robots and continuum robots from the large number of DOF's, that in some cases are redundant, and by the lack of an elastic backbone running down its length. In most of these proposed mechanisms, intrinsic motor actuation, distributed along the robots length, directly drives individual joints within the modules. The kinematics of these mechanisms can be modeled using conventional rigid body techniques; however, its structural design is undesirable for a robotic tail since individual joint actuation is unrequired and the distributed actuators about the robots length will increase the tail mass (as opposed to letting it remain a design variable.) Theoretically, one actuator and speed reduction unit can be mounted on joint along the length of the robotic tail. However, this approach increases actuator loads and results in a heavy, bulky system.

Based on the overview of continuum and serpentine robotics, a modular serpentine structure is chosen to implement the robotic tail for a number of reasons: (1) to preserve the rigid-link structure to simplify modeling, sensing and implementation of the design as opposed to an elastic core continuum robotic structures, (2) To enable spatial tail motions that can generate spatial forces and moments as opposed to existing tails designs that resemble planar, single DOF pendulums, (3) Its modular design enables tails to be interconnected in series to achieve multiple curvatures for increased articulation. A tendon drive transmission system consisting of routed cables will be used to actuate the tail curvature DOF for the following reasons: (1) All actuators can be installed in the base of the tail, and (2) a well-designed cable transmission has little backlash. These merits have made tendon transmission systems well suited for compact, lightweight and high speed applications as demonstrated in robotic hand designs [26, 27].

3 MECHANICAL DESIGN

This section presents the mechanical design of the proposed mechanism where design specifications of a robotic tail discussed in Section 2 are integrated into the DMST.

Fig. 1 presents the detailed mechanical design of the DMST. The overall structure of the tail is constructed from rigid segments that are connected to one another through revolute joints that permit pure relative rotation between neighboring segments. The DMST is composed of two main subsystems: An actuation unit and a scalable series of interconnected tail segments. The two DOF mechanism is capable of roll and tail curvature motions that are actuated using high torque servo motors. Roll and tail curvature motions are actuated using direct drive and a cable transmission system respectfully. The motors and the multi-diameter pulley, that produces cable displacements, are located within the actuator unit that serves as a protective, rigid housing for electrical components and distributes routed cables to their respective pulley diameters at fixed angles.

Segments of the tail are designed to be light weight, rigid structures that are capable of tolerating high loads produced from cable tension. They either have a grooved, semi-circular surface that act as pulleys, used for cable routing, or a flat end surface depending on the location of each segment about the tail length. Flat end segments are placed at the end of the tail and are designed with a hole-pattern that matches the connector port located at the base of the DMST (Fig. 1) to enable the serial connection of modular robotic systems. For example, if a robotic tail is required to have multiple spatial tail curvatures, a sequence of DMSTs can be connected in series by attaching the top segment with the connector port of a separate DMST. If the tail is performing manipulation tasks, modular end effectors, such as a robotic gripper, can be attached to the top segment via a connector port to further increase the tails capabilities as shown in Fig.2.

Each segment is connected on opposing sides via two low friction, nylon coated cables to a single channel of the multi-

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diameter pulley using swaged ball bearings. The pulley consists of 5 grooved channels that are used for routing and terminating cables; thus, can connect up to 5 tail segments. The various dimensions of the pulley diameters cause different cable displacements when rotated that correspond to the formation of tail curvatures. Therefore, a discretized tail curvature can be achieved through the rotation of a single DOF. Channels located within the tail segments, near the termination points of the cables, ensure that the cables are always routed along the same orientation as its respective segment and tangential to its center of rotation – revolute joint. Since cables are routed through segments (visible in Fig. 1), kinematic coupling results due to additional cable displacements caused by the rotation of segments between the termination points.

To ensure a straight home configuration and proper operation of the DMST with minimal backlash/dead-zones, tensioning of the cables is required. This is achieved through a tensioning system built into each side of a segment that consists of a sliding unit that is adjusted using a screw and nut shown in Fig. 1. Tightening of the screw translates the sliding unit inward. With sufficient translation, the sliding unit engages and begins tensioning the routed cables that corresponds an angular adjustment of the segment. Cable tensioning is performed on both sides of each segment to properly align its orientation. Due to kinematic coupling, this tensioning procedure is initiated at the lowest segment and ends at the highest since the rotation of segments near the base affects the orientation of higher located segments.

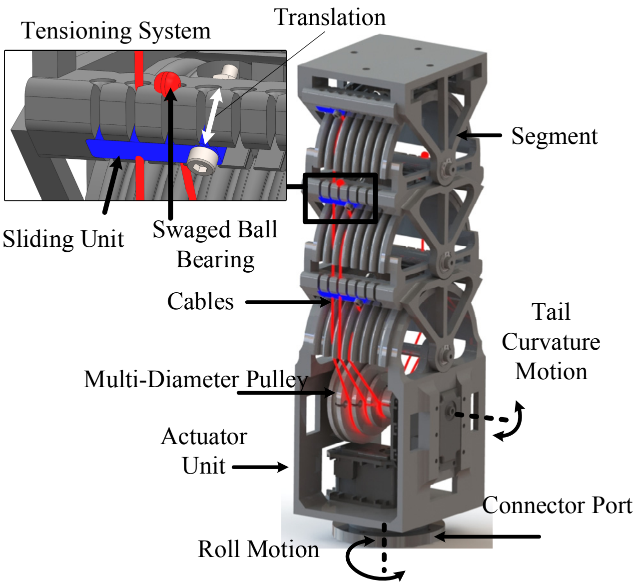

Figure 1. Mechanical design of the DMST

Fig. 2 illustrates an example of a robotic tail design concept constructed from two DMST's connected in series that is attached to a mobile legged robot, in this case a Quadruped experiencing a tip-over motion caused by an external disturbance. This concept illustrates the tail performing a compensating tail motion with two distinct tail curvatures (1 & 2) to generate the required forces and moments to maintain stability of the robot. Once the quadruped is stable, the tail can be used to manipulate its environment by wraping around and grasping onto objects. Due to its modular design, manipulation modules can also be incorporating into the tail to further enhance manipulation capabilities. Figure 2 depicts a gripper module attached to the tip of the tail, this also provides an additional benefit of increased tail inertia to amplify dynamic forces imparted onto the legged robot. However, to generate these dynamic forces, the mechanisms kinematics, governing equations of motion and torque requirements must be determined to properly design the DMST that will be the main focus of upcoming sections in this paper.

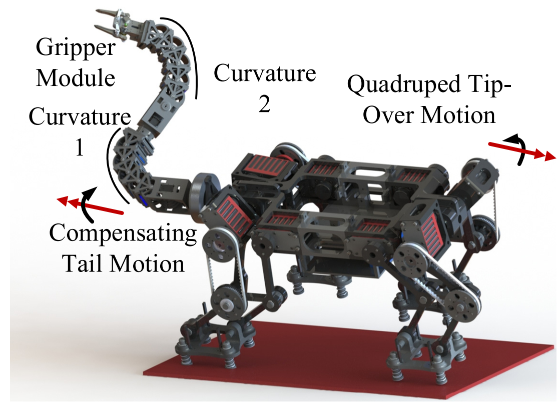

Figure 2. Design concept of two series connected DMST's attached to a quadruped robot

4 FORWARD KINEMATIC ANALYSIS

Forward Kinematic analysis of tendon-driven mechanisms is similar to that of gear driven mechanisms. The analysis can be accomplished in two fundamental steps. The first step involves determining the kinematic relationship between joint angles and cable displacements. The second step involves determining the relationship between pose, position and orientation, of the end effector and joint angles of the open-loop chain. Forward kinematic analysis is first performed on a 2-segment DMST and then is extended to an *N*-segment tail.

The schematic diagram of the 2-segment DMST is shown in Fig. 3. The DMST consists of influencing and dependent segments. An influencing segment is defined as a segment with cables routed through it belonging to other segments. Kinematic coupling results when an influencing segment undergoes angular rotation. Segment 1 is considered to be an influencing segment since segment 2 cables are routed through it. Dependent segments are defined as segments whose final configurations are modified due to kinematic coupling. Segment 1 is not a dependent segment since it is connected directly to a fixed base and its cables are not routed through other segments. Segment 2 is considered a dependent segment since routed cables undergo cable displacement due to angular rotation of segment 1. This results in additional angular rotation of segment 2 that must be considered in computing final tail configuration.

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We first analyze tail curvature kinematics to determine both the kinematic coupling between the segments and the influence between the ratio of segment lengths and pulley diameters. The two-segment DMST is considered as an open loop robotic manipulator with orientation of each segment defined as follows: $\theta_1=\mu_1(r_1, r_2, R_1, R_2)\theta_P$ and $\theta_2=\mu_2(r_1, r_2, R_1, R_2)\theta_P$. Where θ_P is the input angle of the multi-diameter pulley, $\{\mu_1, \mu_2\}$ are scalar ratios dependent on $\{r_1, r_2\}$ that represent pulley radii attached to segment 1 and 2 respectively, and $\{R_1, R_2\}$ the lengths of the segments. These scalar ratios also relate segment angular velocities and accelerations with $\dot{\theta}_P$ and $\ddot{\theta}_P$.

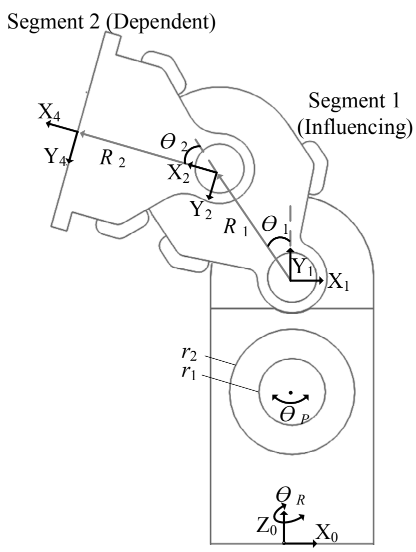

Figure 3. Schematic diagram of the 2-segment DMST

Kinematic analysis to determine the orientation of each segment requires an iterative process due to inherent kinematic coupling of the mechanism. Initial orientations of each segment are first computed due to rotation of the multi-diameter pulley. These values are then modified according to cable displacements caused by angular rotations of influencing segments. The general approach to kinematic analysis is outlined in four main steps:

1) Calculate input angular rotation of all segments due to input angle of the multi-diameter pulley

2) Calculate cable displacement of influencing segments.

3) Convert cable displacements of influencing segments into 'additional angular rotations' of dependent segments.

4) Subtract additional angular rotation from 'input angular rotations' of dependent segments to obtain final orientation of the tail segments.

For the 2-segment DMST shown in Fig. 3, input angular rotation of segments 1 and 2 due to input angle can be calculated by the following relations: $\theta_{1,in}=(r_1/R_1)\theta_P$ and $\theta_{2,in}=(r_2/R_2)\theta_P$.

The final orientation of segment 1 is calculated directly $\theta_1=\theta_{1,in}$ since it is not a dependent segment. Cable displacement caused by segment 1 rotation is then calculated by $s_1=R_1\theta_1$.

Additional angular rotation of segment 2 caused by segment 1 cable displacement, denoted as $\theta_{2,1}$, can be computed by using the length of dependent segments (segment 2) using the relation $\theta_{2,1}=s_1/R_2$.

To compensate for additional angular rotation, the value of $\theta_{2,1}$ is subtracted from input angular rotation resulting in a final orientation of segment 2 equal to $\theta_2= \theta_{2,\text{in}} -\theta_{2,1}$. For a DMST designed with equal length segments $R_1=R_2$, this relation is simplified where $\theta_{2,1}= \theta_1$. This relation physically makes sense since final relative orientation of segment 2 requires subtraction of angular rotation of segment 1 when segment are of equal lengths.

With known geometries and orientations of each segment with a tail curvature, forward kinematics of the DMST is calculated based on the Denavit-Hartenberg (DH) convention [28]. When assigning the coordinate frames according to the convention as seen in Fig. 3, it is possible to calculate the forward kinematics using the homogenous transformation A_{i+1}^{i}

$$A_{i+1}^{i} = \begin{pmatrix} \cos q_i & -\sin q_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin q_i & \cos q_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

Using Eq. 1 along with the DH parameters, the forward kinematics relating the pose, position and orientation of each segment with respect to the fixed coordinate frame X_0Z_0 is a series of homogeneous matrix multiplications.

$$A_4^0 = A_1^0 A_2^1 A_3^2 A_4^3 \quad (2)$$

Kinematic analysis of an *N*-segment DMST follows the same general iterative approach discussed earlier. Initial configurations of *N*-segments are calculated according to input angle of the multi-diameter pulley; then, are modified according to cable displacements caused by additional angular rotations of dependent segments due to inherent kinematic coupling. The equations governing kinematics of an *N*-segment DMST are computed using the iterative algorithm given below:

$$\theta_{i,in} = \frac{r_i}{R_i}\theta_P \quad i = 1:N$$

$$\theta_1 = \theta_{i,in}$$

For i=2:N

$$s_{i-1} = R_{i-1}\theta_{i-1}$$

For k=1:N-1 & $k<i$

$$\theta_{i,k} = \frac{s_k}{R_i}$$

end

$$\theta_i = \theta_{i,in} - \sum_{k=1}^{i-1}\theta_{i,k} = \eta_i\theta_p$$

end

where the variables $\theta_{i,in}$, s_i, $\theta_{i,k}$,θ_i respectively denote i^{th} segment input angular rotation, cable displacement due to

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angular rotation of the i^{th} segment, additional angular rotation of the dependent i^{th} segment due to angular rotation of the influencing segment k, and the final orientation of the i^{th} segment. With known geometries of the mechanism and computed orientations of all the segments, a similar approach may be followed to assign coordinate frames, construct the corresponding DH table and compute the forward kinematics for an N-segment DMST using a series of homogeneous matrix multiplications.

5 DYNAMIC ANALYSIS

In this section dynamic analysis is performed on an N-segment DMST and the equations of motion (EOM) of the system are formulated. Roll motion is directly actuated using a servo motor; however, tail curvature is a resultant of cable tension pulling the segments. Therefore the EOMs are modified to account for this type of actuation. These equations are then used to compute motor torque requirements. Since the DMST consists of rigid bodies interconnected using conventional revolute joints, the EOM can be derived using either Newton-Euler or the Lagrangian method [28]. For our purpose it is more convenient to use the Lagrangian technique.

Although the N-segment DMST is a two DOF mechanism, generalized coordinates must be chosen to define the configuration of each rigid body relative to some reference coordinate frame. Therefore, the generalized coordinates, $\gamma \in R^{N+1}$, are selected to be to be the angular rotation of each rigid body $\gamma=[\theta_R, \theta_1, \theta_2, \ldots, \theta_N]$.

We then define the total kinetic and potential energy of the system to be K and P respectively.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\gamma}_j} - \frac{\partial L}{\partial \gamma_j} = \tau_j,\ j = 1,\ldots,N+1 \tag{3}$$

where $L=K\text{-}P$ is the lagrangian, τ is the torque input and subscript j denotes the j-th generalized coordinate. Expanding Eq. 3 results in the EOM governing the dynamics of the system that can be represented in matrix form shown below

$$M\ddot{\gamma} + C\dot{\gamma} + g(\gamma) = \tau_j = \begin{bmatrix} \tau_R \\ \tau_N \end{bmatrix} \tag{4}$$

where M represents the inertia matrix, C is the matrix of damping coefficients and $g(\gamma)$ is a function of gravitational terms. Input to the system is in the form of torque, τ_R and τ_N, from the actuated roll and tail curvature DOF respectively. EOMs pertaining to tail curvature require modification since motion is controlled using cable tension as opposed to the roll DOF that is directly driven with a motor. To better visualize this, Fig. 4 provides a simplified schematic diagram of an N-segment DMST. Cable tensions are denoted as F_N. The subscript, N, indicates which segment and diameter pulley the cable is connected to. Cables are routed through segments and are directed along the same orientation as its respective segment due to their method of channeling as explained in Section 2. Cables are then redirected to their corresponding pulley diameters at an angle β_N. Since cables are nylon coated to reduce friction, it is assumed in this analysis that cable friction is negligible, cable tensions are constant throughout the cable's entire length and that only one side is under tension during motion for each respective segment while the other side is slack, i.e. has no cable tension. This however may change during a prescribed motion depending on the type of loading as will be further elaborated later on in this section. Therefore, we can express the torque input for each segment as

$$\tau_N = F_N R_N \sin\theta_{TN} \tag{5}$$

where F_N represents the input cable tension causing motion of the segments R_N is the vector oriented from the N-th revolute joint to the point where its cable is fixed, and θ_{TN} is the transmission angle between R_N and its cable tension that varies as the tail changes configuration.

Using the dot product relation, input cable tensions can then be converted back into torque increments, $\tilde{\tau}_N$, using the following relation.

$$F_N = \frac{\tilde{\tau}_N}{r_N \cos\beta_N} \tag{6}$$

Substituting Eqs. 5 and 6 into Eq. 4 yield the modified equations of motion of the system.

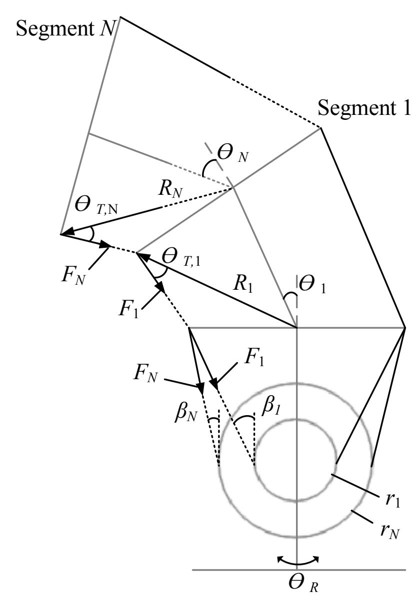

Figure 4. Simplified schematic diagram of an *N*-segment DMST

5.1 Required Torque Analysis

A desired functionality of the DMST would be to generate forces and moments to improve mobile robotic systems performance in terms of stabilization, maneuverability and dynamic self-righting. This is to be achieved using high speed

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tail actuations where the tail rapidly changes its configuration to a given pose. To achieve this type of motion, the actuators must be sized properly to ensure the tail can perform such motions. In this section, the modified EOMs derived in the previous section are used to calculate torque requirements to select motor specifications for a 2-segment DMST.

An example of a single high speed tail actuation would be to rapidly change the angular rotation of the roll and multi-diameter pulley controlling the two DOFs of the system simultaneously from its home configuration to $\pi/2$ rads over a duration of 0.5 seconds with zero angular velocity and acceleration at its start and end pose. The plot of Fig. 5 shows input trajectory profiles of roll and pulley rotation vs time.

Substituting trajectory profiles and mass parameters of the mechanism estimated using CAD into the modified equations of motion for pulley diameters $\{r_1, r_2\} = \{1.75, 3\}$ cm and segment length $\{R_1, R_2\} = \{6, 6\}$ cm, the torque requirements of the system can be calculated. Fig. 6 presents the plot of calculated torque requirements of a 2-segment DMST with a 1 kg tip mass attached to segment 2 when performing defined trajectory profiles.

We notice that torque plots take on a similar curvature in comparison to input angular acceleration due to their direct correlation with one another. incremental torques of the two segments, in the tail curvature plot, initially take on positive values to overcome inertia to move the tail in the positive orientation then change to negative values at approximately 0.14 s and remain offset in this region to oppose gravitational loading. Negative valued torques indicate that the cables tensions have changed sides. At this time, the cable on the right hand side become tensioned while left side becomes slack. It is also noticeable that $\tilde{\tau}_2 > \tilde{\tau}_1$, this is expected since cable tensions of segment 2 act on a larger pulley diameter in comparison those of segment 1; therefore, produces a larger torque about the multi-diameter pulley.

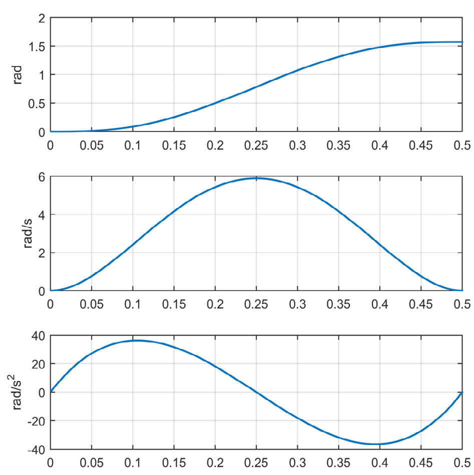

Figure 5. Input trajectory profiles of roll and pulley rotation vs time

The maximum expected torque the roll actuator must supply is equal to 5 Nm. The total torque requirement for the pulley DOF is equal to the addition of maximum incremental torque values; therefore, we expect the pulley actuator to provide 1 Nm.

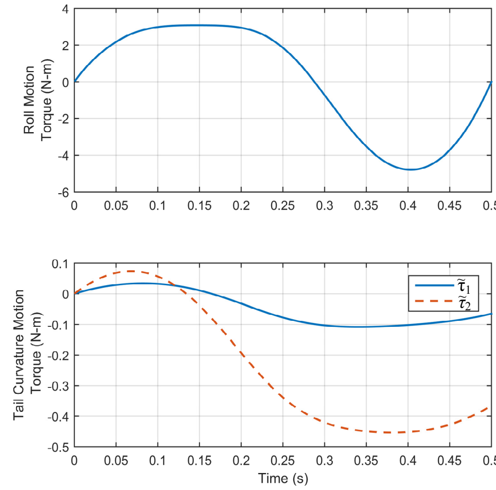

Figure 6. Plot of calculated torque requirements of a 2-segment DMST with 1 kg tip mass

6 OPTIMAL KINEMATIC SYNTHESIS

Kinematic synthesis, sometimes referred to as the Geometric Design Problem [29], involves determining physical dimensions of the DMST from desired, user-defined tail curvatures. Kinematic synthesis may be divided into three main categories: function generation, path generation and motion generation [30]. Precision Point Synthesis is one such technique to synthesize a mechanism that involves selecting a number of precision points and solving a closed form set of equations resulting in mechanism dimensions. The resulting mechanism is expected to perform the desired task with little or no error at the precision points. Selecting more precision points corresponds to better performance; however, this leads to computational difficulties due to solving a set of equations that may be nonlinear with the possibility that a unique solution may not exist. These difficulties create challenges for using conventional techniques of kinematic synthesis for the DMST due to the requirement of a large number of precision points for each segment at multiple desired tail curvatures.

Kinematic synthesis of the DMST involves determining physical dimensions of segment lengths (R_i) and radii (r_i) of the multi-diameter pulley under constraints representing physical limitations of the system. For a large number of segments or multiple DMST's connected in series, the synthesis problem may become difficult to solve. In this analysis, we opt for a multi-objective optimality approach to synthesize the DMST since it enables the formulation of design problem with heterogeneous cost functions and constraints (geometric, kinematic equalities and inequalities). This formulation aims at reducing the complexities of solving exact closed form set of

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equations with a large number of precision points and tail curvatures.

We start off by defining a number of desired precision point's (n) describing the desired position (P) and orientation (Q) for each segment within the DMST for a single tail configuration. This process is repeated for m desired tail curvatures. This results in a total of $n \times m$ precision points that will be used for kinematic synthesis. A cost function is then formulated to optimize the discrepancy, e, between the desired precision points and the actual position ($\underline{P}$) and orientation ($\underline{Q}$) of each segment computed from forward kinematic relations. The overall optimal design problem may be expressed in the form of the constrained problem as:

$$\min_{R_i, r_i \in \Omega} e = \sum_{k=1}^{m} \sum_{i=1}^{n} (\sigma_1 (P_{i,k} - \underline{P}_{i,k})^2 + \sigma_2 (Q_{i,k} - \underline{Q}_{i,k})^2)$$

Subject to the set of geometric constraints

$$\Omega = \begin{cases} e \leq \delta \\ R_{i,min} \leq R_i \leq R_{i,max} \\ r_{i,min} \leq r_i \leq r_{i,max} \end{cases} \quad (7)$$

where $P_{i,k}$ and $Q_{i,k}$ represent the i-th precision point, position and orientation respectfully, at the k-th tail curvature, σ_1 and σ_2 are scalar values to make discrepancies dimensionless, δ is the error threshold and $\{R_{i,min}, R_{i,max}, r_{i,min}, r_{i,max}\}$ define the bounds of the allowable range of values of R_i and r_i. Solving Eq. 7 yields an optimal solution of the mechanism's physical dimensions.

6.1 Optimal Solution: Case study

In this section, a design case study for a 2-segment DMST is presented based on the proposed optimal kinematic synthesis approach. The objective of this optimization is to synthesis a DMST mechanism to minimize the variation between two desired curvatures that represent the final tail curvature after the multi-diameter pulley rotates by 90°. In this case study, n=2 and m=2 represent the two sets of precision points for the two segments and two desired tail curvatures respectfully. Table 1 presents the set of desired precision points used for kinematic synthesis. These values were then input into the optimal design equation with physical dimension constraint parameters equal to $\{R_{1,min}, R_{2,min}, r_{1,min}, r_{2,min}\}=\{3, 3, 0.5, 1\}$ cm, and $\{R_{1,max}, R_{2,max}, r_{1,max}, r_{2,max}\}=\{7, 7, 1, 2\}$ cm. Eq. 7 was then solved using Interior point convex optimization [31].

The design problem converged to a solution representing dimensions of the synthesized mechanism equivalent to $\{R_1, R_2, r_1, r_2\} = \{5.9, 4.7, .75, 1\}$ cm. As a comparison, optimal mechanism dimensions were used to compute precision points for the optimal configuration after the multi-diameter pulley rotates by 90° (shown in Table 1). It is noticeable that the optimized mechanism's provides an optimal solution between two desired tail curvatures since it's computed precision point's falls approximately in between those of configurations 1 and 2. In addition, optimal mechanism dimensions satisfy physical dimension constraints.

Table 1. Precision points for optimal kinematic synthesis

		Curvature 1	Curvature 2	Optimal Curvature
Seg. 1		$P_{1,1}$=[-2, 10]	$P_{1,2}$=[-1,10]	$\underline{P}_1$=[-1.1,9.8]
		$Q_{1,1}$ = [17]	$Q_{1,2}$ = [8]	$\underline{Q}_1$ =[11.5]
Seg. 2		$P_{2,1}$= [-6,14]	$P_{2,2}$ = [-3,15]	$\underline{P}_2$ =[-2.8,14.2]
		$Q_{2,1}$= [33]	$Q_{2,2}$ = [8]	$\underline{Q}_2$ =[3.55]

Units: $P_{i,k}$ (cm), and $Q_{i,k}$ (°)

7 CONCLUSION

This paper presents the design and analysis of a novel Discrete Modular Serpentine Tail for mobile legged robotic systems. The proposed mechanism is envisioned to be attached on board these robotic systems to enhance performance in terms of stabilization, maneuverability and manipulation. The mechanical design enables spatial tail motions and the capability of forming articulate configurations. The analysis presented in this paper formulates the forward kinematic relations and the torque requirements of a kinematically coupled robotic mechanism utilizing hybrid actuation in the form of direct drive and a cable transmission system for roll and tail curvatures respectfully. Optimal kinematic synthesis enables the computation of the proposed mechanism's segment lengths and multi-diameter pulley dimensions to obtain a best-fit of user defined tail configurations subject to geometric constraints.

Ongoing research involves formulating a more accurate dynamic model to better approximate torque requirements of the system by considering routed cable friction. Further analysis will be performed to determine joint forces generated from dynamic tail motions to develop an analytical model for computing loading profiles that the tail can transfer through base. In addition, loading profile analysis will be performed to determine the impact of varying design parameters such as tail trajectories, coupling ratios and mass distribution properties. Multi-body dynamic simulations and experimentation using an integrated prototype and hardware in the loop setup will then be used to analyze the performance improvements the mechanism can provide legged robots and will be used to further validate kinematics and dynamic models.

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